S.No. 3147 **12PMA13**

(For the candidates admitted from 2012–2013 onwards)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2020.

Fourth Semester

Mathematics

PROBABILITY THEORY

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. State the Bayes theorem.

2. Define Marginal distribution.

3. If X is a random variable with the density function $f(x)=\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$, show that the expected value of the random variable X is zero.

4. State the Lapunov inequality.

5. Find the semi invariance of the Poisson distribution.

6. Define probability generating function.

7. Define Hypergeometric distribution.

8. Prove that the sum of two independent Poisson random variable is a Poisson random variable.

9. State Borel-Cantelli theorem.

10. State Kolmogorov inequality.

SECTION B — ($5 \times 5 = 25$ marks)

Answer ALL the questions.

11. (a) If F is the distribution function of one dimensional random variable X, then prove that

(i) $0 \leq F(x) \leq 1$ and

(ii) $F(x) < F(y)$ if $x < y$.

Or

(b) Let X be a random variable with distribution function $F(X)$, then find the distribution function of $Y = aX + b$.

12. (a) Define the following :

(i) Absolute moment;

(ii) Regression of second type.

Or

(b) Show that the relation $P(Y = aX + b) = 1$ is true if $\rho^2 = 1$.

13. (a) Obtain the formula for moments if the characteristic function of a random variable is given.

Or

(b) Find the probability generating function of the Poisson distribution.

14. (a) Find the mean and variance of Binomial distribution.

Or

(b) Find the value of A, if the density function is $f(x) = \frac{A}{x^2 - 2x + 5}, \ -\infty < x < \infty$.

15. (a) State and prove Kolmogorov inequality.

Or

(b) If $X_n \to p X$, then prove that $X_n \to_L X$.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Let $\{A_n\}$, $n = 1, 2, \cdots$ be a non increasing sequence of events and let A be their product. Then prove that $P(A) = \lim_{n \to \infty} P(A_n)$.

17. Find the marginal distribution of the random variables X and Y given that $f(x, y) = \frac{1}{2\pi} e^{-\frac{x^2+2xy+y^2}{2}}$, $-\infty < x, y < \infty$. Also find the correlation coefficient.

18. Let $F(x)$ and $\psi(x)$ be the distribution function and the characteristic function, respectively of the random variable X. If $a+h$ and $a-hh>0$ be arbitrary points of continuity of $F(X)$, then prove that $F(a+h) - F(a-h) = \lim_{T \to \infty} \frac{1}{\pi} \int_{-T}^{T} \frac{\sinh t}{t} e^{-ita} \psi(t)\, dt$.

19. If $X \sim N(1, 2)$, find the probability that X is greater than 3 in absolute value.

20. State and prove Lindeberg-Levy Central Limit Theorem.
