S.No. 3151 **17PMA13**

(For the candidates admitted from 2017–2018 onwards)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2020.

Fourth Semester

Mathematics

PROBABILITY THEORY

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define impossible event.

2. Define marginal distribution.

3. Define absolute moments.

4. Define co-efficient of skewness.

5. Define probability generating function.

6. Show that $\phi(0)=1$.

7. Write the probability function of Cauchy distribution.

8. Write the relation between beta and gamma function.

9. State Levy-Cramer Theorem.

10. Define stochasticallv convergent.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) State and prove Bayes theorem.

Or

(b) Obtain the distribution function for

$$f(x)=\begin{cases}0 & for \quad x<0 \\ \frac{x}{2} & for \quad 0 \le x \le 2 \\ 0 & for \quad x>2\end{cases}.$$

12. (a) State and prove Tchebyshev inequality.

Or

(b) The random variable X can take on two values 2 and 4, where $P(X=2)=0.2$ and $P(X=4)=0.8$. Find $E(X^2)$.

13. (a) Find the density function of a random variable X is given by the formula $\phi(t) = e^{\frac{-t^2}{2}}$.

Or

(b) Find the semi- invariants of the Poisson distribution.

14. (a) Obtain the mean and variance of binomial distribution.

Or

(b) Find the mean and variance of the beta distribution.

15. (a) State and prove weak law of large number.

Or

(b) Let $F_n(x), (n = 1,2,3,...)$ be the distribution function of the random variable X_n. Show that the sequence $\{X_n\}$ is stochastically convergent to zero if and only if $\{F_n(x)\}$ satisfies the relation

$$\underset{n\to\infty}{Lt}\, F_n(x) = \begin{cases} 0 & for \quad x \le 0 \\ 1 & for \quad x > 0 \end{cases}.$$

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. If $\{A_n\} n = 1,2,3 \ldots (X,Y)$ be a non-decreasing sequence of events and let A be their alternative then prove that $P(A) = \lim_{n \to \infty} P(A_n)$.

17. State and prove Lapunov inequality.

18. If (X,Y) is a two-dimensional random variable with joint distribution with $f(x) = \begin{cases} \frac{1}{4}\left[1 + xy\left(x^2 - y^2\right)\right] & for \quad |x| \le 1 and |y| \le 1 \\ 0 & for \quad otherwise \end{cases}$.

Test whether X and Y are not independent and also find density function of sum of the two random variable.

19. Derive the Poisson distribution from the binomial distribution.

20. State and prove De-Moivre Laplace theorem.

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