(8 pages)

S.No. 3041

19PMA08

(For the candidates admitted from 2019 – 2020 onwards)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2020.

Third Semester

Mathematics

PARTIAL DIFFERENTIAL EQUATION

Time : Three hours Maximum : 75 marks

SECTION A — (15 × 1 = 15 marks)

Answer ALL questions.

1. The operator $F(D, D')$ is said to be reducible if it can be factorized into the linear factor of the type

   (a) $D + aD' + b$ (b) $D + aD'$

   (c) $a + b$ (d) D

2. The auxiliary equation has equal roots, then the complementary function is

   (a) $z = f_1(y + mx) + xf_2(y + mx)$

   (b) $z = f_1(y + mx) + f_2(y + mx)$

   (c) $z = f_1(y + mx) + f_2(x + mx)$

   (d) $z = f_1(y + mx) + yf_2(x + mx)$

3. A second order partial differential equation is said to be parabolic if

(a) $B^2 - 4AC = 0$ (b) $B^2 - 4AC < 0$

(c) $B^2 - 4AC > 0$ (d) $c^2 - 4AB = 0$

4. Two dimensional Laplace equation is

(a) $\nabla^2 U > 0$ (b) $\nabla^2 U < 0$

(c) $\nabla^2 U = 0$ (d) $\nabla U = 0$

5. Newtons law of gravitation

(a) $F = \frac{Gm_1 m_2}{r^2}$ (b) $F = \frac{m_1 m_2}{r^2}$

(c) $F = \frac{Gm}{r^2}$ (d) $F = \frac{Gm_1 m_2}{r}$

6. The auxiliary equation has un equal roots, then the complementary function is

(a) $z = f_1(y + m_1 x) + x f_2(y + m_2 x)$

(b) $z = f_1(y + m_1 x) + f_2(y + m_2 x)$

(c) $z = f_1(y + m_1 x) + y f_2(x + m_2 x)$

(d) $z = f_1(y + mx) + f_2(x + mx)$

7. Suitable solution for one dimentional heat equation is __________

(a) $T(x,t) = (A\cos\alpha x + B\sin\alpha x)^{e^{-\alpha^2 t}}$

(b) $T(x,t) = (A\cos\alpha x + B\sin\alpha x)^{e^{\alpha^2 t}}$

(c) $T(x,t) = (A\cos\alpha x + B\sin\alpha x)$

(d) $T(x,t) = (Ax + B)^{e^{-\alpha^2 t}}$

8. Possible solution of heat equation when $\lambda = 0$ is _________

(a) $T(x,t) = (Ax + B)e^{-\alpha^2 t}$

(b) $T(x,t) = (Ax + B)e^{\alpha^2 t}$

(c) $T(x,t) = (Ax + B)e^{t}$

(d) $T(x,t) = (Ax + B)$

9. Insulated boundary conditions which states that heat flow across the surface is

(a) 0

(b) π

(c) 1

(d) λ

10. The wave equation is of the form

(a) $\frac{\partial^2 u}{\partial t^2} = c^2 \nabla^2 u$ (b) $\frac{\partial^2 u}{\partial t^2} = \nabla^2 u$

(c) $\frac{\partial u}{\partial t} = c^2 \nabla^2$ (d) $\frac{\partial^2 u}{\partial t^2} = c^2$

11. In one dimentional wave equation the slope of the deflection curve is _____

(a) large (b) small

(c) zero (d) none

12. The solution of wave equation are called ______ function.

(a) wave (b) string

(c) heat (d) none

13. If f(t) is said to be of exponential order a, if there exist a real and finite positive number M such that ______________

(a) $\lim_{t \to \infty} |f(t)| e^{-\alpha t} \leq M$

(b) $\lim_{t \to \infty} |f(t)| e^{\alpha t} \leq M$

(c) $\lim_{t \to 0} |f(t)| e^{-\alpha t} \leq M$

(d) $\lim_{t \to \infty} |f(t)| e^{-\alpha t} = M$

14. Laplace transform of $f(t)$ is defined by $L(f(t))$ = ___________

(a) $\int_0^\infty f(t)e^{-st}dt$ (b) $\int_0^\infty f(t)e^{st}dt$

(c) $\int_{-\infty}^\infty f(t)e^{-st}dt$ (d) $\int_0^\infty f(s)e^{-st}ds$

15. $L(1) =$ ___________

(a) $1/t$ (b) $1/s$

(c) $1/a$ (d) 0

SECTION B — (2 × 5 = 10 marks)

Answer any TWO questions.

16. If $u = f(x+iy) + g(x+iy)$, where f and g are arbitrary function. Show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.

17. Show that the two dimension laplace equation $\nabla^2 v = 0$, in the plane polar coordinate r and θ has the solution of the form $(Ar^n + Br^{-n})e^{\pm in\theta}$.

18. Derive possible solutions of one Dimensional heat equation.

19. State the assumptions for derivations of one dimensional wave equation.

20. Find $L(Sinh\, at)$.

SECTION C — (5 × 10 = 50 marks)

Answer ALL questions.

21. (a) Solve the equation

$$\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} - \frac{\partial^3 z}{\partial x \partial y^2} + 2\frac{\partial^3 z}{\partial y^3} = e^{x+y}.$$

Or

(b) Reduce the equation

$$(n-1)^2 \frac{\partial^2 z}{\partial x^2} - y^{2n}\frac{\partial^2 z}{\partial y^2} - \frac{\partial^3 z}{\partial x \partial y^2} = ny^{2n-1}\frac{\partial z}{\partial y}$$ to

canonical form and find its general solution.

22. (a) Derive solution for Laplace equation in cylindrical coordinates.

Or

(b) Derive interior Neumann problem for a circle.

23. (a) Show that the solution of equation $\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2}$ satisfying the conditions

(i) $T \to 0, as\, t \to \infty$

(ii) $T = 0$ for $x = 0$ and $x = a$ for all $t > 0$

(iii) $T = x$ when $t = 0$ and $0 < x < a$ is $T(x,t) = \frac{2a}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} \sin\left(\frac{n\pi x}{a}\right)$ exp $\left(-\left(\frac{n\pi}{a}\right)^2 t\right)$.

Or

(b) Find the temperature in a sphere of radius a when its surface is maintained at zero temperature and its initial temperature is $f(r,\theta)$.

24. (a) Obtain the solution of the radio equation $\frac{\partial^2 v}{\partial x^2} = LC \frac{\partial^2 v}{\partial t^2}$ appropriate to the case when a periodic e.m.f. $v_0 \cos pt$ is applied at the end $x = 0$ of the line.

Or

(b) A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a Position given by $y - y_0 \sin^3\left(\frac{\pi x}{l}\right)$. It released from rest from this position, find the displacement $y(x,t)$.

25. (a) Solve the initial boundary value problem $\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}, 0 < x < 1, t > 0,$ subject to initial and boundary conditions

$u(x,0) = \sin \pi x, \frac{\partial u}{\partial t}(x,0) = -\sin \pi x, 0 < x < 1$ and $u(0,t) = u(1,t) = 0, t > 0$.

Or

(b) Displacement of a infinite string is governed by $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, -\infty < x < \infty$ Subject to initial condition

$u(x,0) = f(x), -\infty < x < \infty, \frac{\partial u}{\partial t}(x,0) = 0.$
