(8 pages)

S.No. 5036

19PMA04

(For the candidates admitted from 2019 – 2020 onwards)

M.Sc. DEGREE EXAMINATION, MARCH/APRIL 2021

First Semester

Mathematics

ORDINARY DIFFERENTIAL EQUATIONS

Time : Three hours Maximum : 75 marks

PART A — (15 × 1 = 15 marks)

Answer ALL questions.

1. The differential equation $L(y) = y'' + a_1 y' + a_2 y = 0$, then $L(e^{rx})$ is

   (a) $r^2 + a_1 r + a_2$
   (b) $(r^2 + a_1 r + a_2)e^{rx}$
   (c) $(r^2 + a_1 r + a_2)y$
   (d) $(r^2 + a_1 r + a_2)e^{-rx}$

2. The differential equation $y'' + a_1 y' + a_2 y = 0$ then every solution tends to zero as

   (a) $a_1 < 0$
   (b) $a_1 = 0$
   (c) $a_1 > 0$
   (d) $a_1 \neq 0$

3. If $y''+a_1 y'+a_2 y$, where the constants a_1, a_2 are real. Suppose $\alpha+i\beta$ is a complex root of the characteristic polynomial, then α, β are

(a) equal (b) complex

(c) real (d) natural

4. Let $L(y)=y''+a_1 y'+a_2 y$, where a_1, a_2 are constants and let p be the characteristic polynomial in

(a) $p(r)=r(r-1)+ar+b$

(b) $p(r)\ r^2+a_1 r+a_2$

(c) $p(r)=(r^2+a_1 r+a_2)e^x$

(d) None of the above

5. The differential equation $y''+W^2 y=A\cos wx$, where A, w are positive constants then every solution φ is such that $|\phi(x)|$ assumes arbitrarily large value as

(a) $x\to\infty$ (b) $x\to 0$

(c) $x\to-\infty$ (d) $x\to-1$

6. Two functions $\phi_1(x)=e^x, \phi_2(x)=e^{-x}$, $\phi_3(x)=e^{2x}$ defined $-\infty<x<\infty$ are

(a) Linearly independent

(b) Linearly dependent

(c) Both (a) and (b)

(d) None of the above

7. Let x_0 be in I and let $\alpha_1,...\alpha_n$ be any n constants there is atmost one solution φ of $L(\varphi)=0$ on I satisfying $\phi(x_0)=\alpha_1, \phi'(x_0)=\alpha_2,...\phi^{(n-1)}(x_0)=\alpha_n$

(a) Uniqueness theorem

(b) Initial value problem

(c) Existence theorem

(d) None of the above

8. let $\alpha_{ij}=\int_a^b \overline{\phi_i(x)\phi_j}(x)dx\,(i,j=1,2,..n)$ then $\phi_1,...\phi_n$ are linearly independent on

(a) $a\leq x\leq b$ (b) $a\geq x\geq b$

(c) $b\leq x\leq a$ (d) $a\leq x\leq 0$

9. If $y''+\alpha(x)\,y=0$, where α is a continuous function on $-\infty<x<\infty$ which is of period $\xi>0$ then

(a) $W(\phi_1,\phi_2)(x)=0$ (b) $W(\phi_1,\phi_2)(x)\neq 0$

(c) $W(\phi_1,\phi_2)(x)=1$ (d) $W(\phi_1,\phi_2)(x)\neq 1$

10. If r_1 and r_2 are the roots of the quadratic $q(r)=r(r-1)+ar+b$ then $q(r)=0$ is called

(a) Characteristic polynomial

(b) Indical polynomial

(c) Legendre polynomial

(d) Bessel's polynomial

11. Let $x^2y''+ax\,y'+by=0$ where a,b are constants. Let $\psi(t)=\phi(e^t)$ then

(a) $\psi(\log x)=-\phi(x)$

(b) $\psi(\log x)=\phi(x)$

(c) $\psi(|\log x|)=|\phi(x)|$

(d) $\psi(|\log x|)=-|\phi(x)|$

12. The differential equation does not have any power series solution at an

(a) Regular singular point

(b) Singular point

(c) Irregular singular point

(d) None of the above

13. The existence and uniqueness of solution to first order equation is

(a) $y' = f(x, y)$ (b) $y' = f(x, y) + c$

(c) $y' = f'(x, y)$ (d) $y' = f''(x, y)$

14. The solution of the problem $y' = 1 + y, y(0) = 0$ then ϕ exists only on

(a) $-\pi < x < \pi$ (b) $-\frac{\pi}{2} < x < \frac{\pi}{2}$

(c) $\frac{\pi}{2} > 0$ (d) $-\infty < x < \infty$

15. If $\phi_k(x) \to \phi_1(x)$ for each x satisfying ——————

(a) $|x| \leq 1$ (b) $|x| \leq -\frac{1}{2}$

(c) $|x| \leq \frac{1}{2}$ (d) $|x| \leq -1$

PART B — (2 × 5 = 10 marks)

Answer any TWO questions.

16. State and prove Uniqueness theorem.

17. Let ϕ be any solution of

$L(y)=y^n+a_1 y^{(n-1)}+\ldots+a_n y=0$ on an interval I containing a point x_0. Then for all x in $I\|\phi(x_0)\|e^{-k|x-x_0|}$ where $k=1+|a_1|+\ldots+|a_n|$.

18. State Existence theorem and Uniqueness theorem of initial value problems for the homogeneous equation.

19. Find the singular points of the following equation and determine those which are regular singular points of $xy''+4y=0$.

20. State existence theorem of convergence of the successive approximations.

PART C — (5 × 10 = 50 marks)

Answer ALL questions.

21. (a) State and prove Existence theorem.

Or

(b) If ϕ_1,ϕ_2 are two solutions of $L(y)=0$ on an interval I containing a point x_0 Then $W(\phi_1,\phi_2)(x)=e^{-a_1(x-x_0)}W(\phi_1,\phi_2)x_0$.

22. (a) Compute the solution of ψ of $y''' + y'' + y' + y = 1$ which satisfies $\psi(0) = 0, \psi'(0) = 1, \psi''(0) = 0$.

Or

(b) Let $\alpha_1, ... \alpha_n$ be any n constants and let x_0 be any real number. There exist a solution ϕ of $L(y) = 0$ on $-\infty < x < \infty$ satisfying $\phi(x_0) = \alpha_1, \phi'(x_0) = \alpha_2, .., \phi^{(n-1)}(x_0) = \alpha_n$.

23. (a) Prove that there exist n linearly independent solutions of $L(y) = 0$ on I.

Or

(b) One solution of $y'' - \frac{2}{x^2} y = 0, 0 < x < \infty$ is $\phi_1(x) = x^2$. Find a basis for the solutions for $0 < x < \infty$.

24. (a) Find all solutions of the equation for $x > 0, x^2 y'' + xy' + y = 0$.

Or

(b) Show that

(i) $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \cos x$.

(ii) $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$.

25. (a) Find all real valued solutions of the equation $y'=3y^{2/3}$.

Or

(b) Let M, N be two real valued functions which has continuous first partial derivatives on same rectangle $R:|x-x_0|\le a;|y-y_0|\le b$. Then the equation $M(x,y)+N(x,y)y'=0$ is exact in R if and only if $\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$ in R.