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S.No. 238

12 PMAZ 01

(For the candidates admitted from 2012–2013 onwards)

M.Sc. DEGREE EXAMINATION, NOVEMBER-2020.

First Semester

Mathematics

Elective — NUMERICAL ANALYSIS

Time : Three hours   Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Using Taylor series method, find $y(0.1)$ if $y' = x^2 + y^2$, $y(0) = 1$.

2. Write the Milne's Predictor and Corrector Formulae.

3. What is the limitation in using Picard's method of successive approximations?

4. Compute y at $x = 0.25$ by Modified Euler's Method given $y' = 2xy$, $y(0) = 1$.

5. Write Runge's formula.

6. Write the formula to solve second order ODE using Runge-Kutta method of fourth order.

7. Define different quotient.

8. Classify the equation :

$$u_{xx} + 4u_{xy} + (x^2 + 4y^2)u_{yy} = \sin x + y .$$

9. Write the implicit formula to solve one dimensional heat flow equation $u_{xx} = \frac{1}{c^2} u_t$.

10. State the explicit scheme to solve the wave equation.

PART B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) Find $y = 0.1(0.1)0.4$, given $\frac{dy}{dx} = x^2 - y$, $y(0) = 1$, using Taylor series method correct to 4 decimal places.

Or

(b) Using Adam's method find $y(0.4)$ given $\frac{dy}{dx} = \frac{1}{2}xy$, $y(0) = 1$ $y(0.1) = 1.01$, $y(0.2) = 1.022$, $y(0.3) = 1.023$.

12. (a) Approximate y and z at $x = 0.1$ using Picard's method to the equation $\frac{dy}{dx} = z$, $\frac{dz}{dx} = x^3(y+z)$, given that $y(0) = 1$ and $z(0) = 0.5$.

Or

(b) Solve numerically $y' = y + e^x$ $y(0) = 0$ for $x = 0.2, 0.4$ by Improved Euler's Method.

13. (a) Using Runge-Kutta Method of fourth order compute $y(0.1)$ given $y' + y + xy^2 = 0$, $y(0) = 1$, correct to 4 decimal places.

Or

(b) Given $y' = x^2 - y$, $y(0) = 1$ find $y(0.1)$ using Runge-Kutta method of third order.

14. (a) Derive the Standard Five Point Formula.

Or

(b) Explain Liebmann's iteration process to solve Laplace's equation.

15. (a) Solve $u_{xx} = 32u_t$, taking $h = 0.25$ for $t > 0$, $0 < x < 1$ and $u(x, 0) = 0$, $u(0, t) = 0$, $u(1, t) = t$ by Bender Schmidt Method.

Or

(b) Solve numerically $4u_{xx} = u_{tt}$, with the boundary conditions $u(0, t) = 0$, $u(4, t) = 0$ and the initial conditions $u_t(x, 0) = 0$ and $u(x, 0) = x(4 - x)$ taking $h = 1$ (for 4 time steps).

PART C — (3 × 10 = 30 marks)

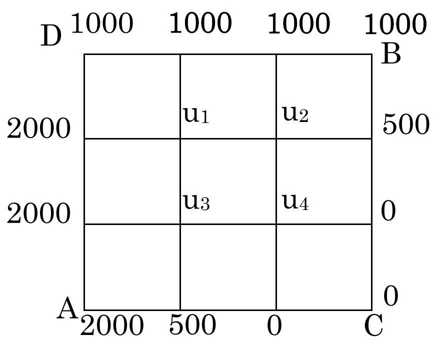
Answer any THREE questions.

16. By using Taylor series, method calculate $y(0.1)$ given $y'' = y + xy'$, $y(0) = 1$, $y'(0) = 0$.

17. Use Picard's method to approximate y when $x = 0.1$ given $\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} + y = 0$ and $y = 0.5$, $\frac{dy}{dx} = 0.1$ when $x = 0$.

18. Find $y(0.1)$ and $z(0.1)$ from the system of equations $y' = x + z$, $z' = x - y^2$ given $y(0) = 2$, $z(0) = 1$ using Runge-Kutta method of fourth order.

19. Evaluate the function $u(x, y)$ satisfying $\nabla^2 u = 0$ at the lattice points given the boundary values as follows :



20. Using Crank-Nicholson Scheme, solve

$u_{xx} = 16 u_t$, $0 < x < 1$, $t > 0$ given $u(x, 0) = 0$, $u(0, t) = 0$, $u(1, t) = 100t$. Compute u for one step in t direction taking $h = \frac{1}{4}$.