(6 pages)

S.No. 330

17PMA06

(For the candidates admitted from 2017 – 2018 onwards)

M.Sc. DEGREE EXAMINATION, APRIL 2019.

Second Semester

Mathematics

FLUID DYNAMICS

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define vorticity vector.

2. Write the general equation of continuity.

3. Write Eulers equation of motion.

4. Write the Bernoulli's equation.

5. Define simple sink.

6. Write Weiss's sphere Theorem.

7. Write Cauchy-Riemann equations.

8. Write the Milne-Thomson Circle Theorem.

9. Write components of stress parallel to the axes.

10. Write the relations between cartesian components of stress.

PART B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) At the point in an incompressible fluid having spherical polar coordinates (r, θ, ψ), the velocity components are $[2Mr^{-3}\cos\theta, Mr^{-3}\sin\theta, 0]$, where M is a constant. Show that the velocity is of potential kind. Also find the velocity potential and the equations of the streamlines.

Or

(b) Liquid flows through a pipe whose surface is the surface of revolution of the curve $y = a + kx^2/a$ about x-axis $(-a \le x \le a)$. If the liquid enters at the end $x = -a$ of the pipe with velocity V. Find the time taken by a liquid particle to traverse the entire length of the pipe from $x = -a$ to $x = a$.

12. (a) Find the thrust on the hemisphere $r = a$, $0 \le \theta \le \frac{1}{2}\pi$.

Or

(b) Prove that at any point P of a moving inviscid fluid, the pressure p is the same in all directions.

13. (a) Doublets of strength μ_1, μ_2 are situated at points A_1, A_2 whose Cartesian coordinates are $(0, 0, c_1)$, $(0, 0, c_2)$, their axes being directed towards and away from the origin respectively. Find the condition that there is no transport of fluid over the surface of the sphere $x^2 + y^2 + z^2 = c_1 c_2$.

Or

(b) Prove that the image of a doublet in an infinite rigid plane is an equal doublet symmetrically disposed with respect to the plane.

14. (a) Discuss the flow for which $w = z^2$.

Or

(b) Find the total complex velocity potential due to a line doublet parallel to the axis of a right circular cylinder.

15. (a) Discuss the translational motion of fluid element.

Or

(b) Discuss the steady motion of a viscous flow between parallel planes.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Test whether the motion specified by

$$q = \frac{k^2(xj - yi)}{x^2 + y^2} \quad (k = \text{constant})$$

is a possible motion for an incompressible fluid. If so, determine the equations of the streamlines. Also test whether the motion is of the potential kind and if so determine the velocity potential.

[P.T.O.]

17. AB is a tube of small uniform bore forming a quadrantal arc of a circle of radius a and centre O, OA being horizontal and OB vertical with B below O. The tube is full of liquid of density ρ, the end B being closed. If B is suddenly opened, show that the pressure at a point whose angular distance from A is θ immediately drops to

$$\rho g a\left(\sin\theta - \frac{2\theta}{\pi}\right)$$

above atmospheric pressure. Prove further that when the liquid remaining in the tube subtends an angle β at the centre,

$$\frac{d^2\beta}{dt^2} = -\frac{2g}{\alpha\beta}\sin^2\left(\frac{\alpha}{\beta}\right)$$

18. A three dimensional doublet of strength μ whose axis is in the direction $\overline{Ox}$ is distant a from the rigid plane $x=0$ which is the sole boundary of liquid of density ρ, infinite in extent. Find the pressure at a point on the boundary distant r from the doublet given that the pressure at infinity is p_∞. Show that the pressure is least at a distance $a\sqrt{5}/2$ from the doublet.

19. A two dimensional doublet of strength μi is at the point $z = ia$ in a stream of velocity $-Vi$ in a semi-infinite liquid of constant density occupying the half plane $y>0$ and having $y=0$ as a liquid boundary (i is the unit vector in the positive x-axis). Show also that, for $0<\mu<4a^2V$, there are no stagnation points on this boundary and that the pressure is minimum at the origin and a maximum at the points $x=\pm a\sqrt{3}$.

20. Derive the Navier Stoke's equation of motion of a viscous fluid.
