(a) $(f+g)'(x) = f'(x) + g'(x)$;

(b) $(fg)'(x) = f'(x)g(x) + f(x)g'(x)$;

(c) $\left(\frac{f}{g}\right)'(x) = \frac{g(x)f'(x) - g'(x)f(x)}{g^2(x)}, g(x) \neq 0$.

18. If α increases monotonically and $\alpha' \in \Re$ on $[a, b]$. Let f be a bounded real function on $[a, b]$. Then $f \in \Re(\alpha)$ if and only if $f\alpha' \in \Re$. In that case prove $\int_a^b f\,d\alpha = \int_a^b f(x)\alpha'(x)\,dx$.

19. Suppose $f_n \to f$ uniformly on a set E in a metric space. Let x be a limit point of E, and suppose that $\lim_{t \to x} f_n(t) = A_n$ ($n = 1, 2, 3, \ldots$). Then prove that $\{A_n\}$ converges, and $\lim_{t \to x} f(t) = \lim_{n \to \infty} A_n$.

20. State and prove Stirling's formula.

(For the candidates admitted from 2012–2013 onwards)

M.Sc. DEGREE EXAMINATION,
NOVEMBER/DECEMBER 2015.

First Semester

Mathematics

REAL ANALYSIS

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define open cover.

2. When $f(x) = \begin{cases} x \ (x \text{ rational}) \\ 0 \ (x \text{ irrational}) \end{cases}$ has continuous and discontinuity of the second kind at every point x.

3. Define Taylor's theorem.

4. When $f(x)$ is monotonically increasing in (a, b).

5. Write equation of Riemann-Stieltjes integrals.

6. Write Holder's inequality and Justify when it will become Schwarz inequality.

7. Define pointwise bounded.

8. State Stone Weierstrass theorem.

9. Define analytic function.

10. Write the relation between beta and gamma function.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) Show that compact subsets of metric spaces are closed.

Or

(b) If f is a continuous mapping of a compact metric space X into a metric space Y then show that $f(X)$ is compact.

12. (a) State and mean value theorem.

Or

(b) State and prove Taylor's theorem.

13. (a) State and prove Holder's inequality.

Or

(b) Prove that $\int_{-a}^{b} f dx \le \int_{a}^{-b} f dx$.

14. (a) State and prove Stone-Weierstrass theorem.

Or

(b) If K is compact if $f_n \in l(K)$ for $n = 1, 2, 3....$ and if $\{f_n\}$ is pointwise bounded and equicontinuous on K then show that $\{f_n\}$ is a uniformly bounded on K.

15. (a) Prove that $\beta(x, y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$.

Or

(b) Given a double sequence $\{a_{ij}\}$, $i = 1, 2, 3, ...,$ $j = 1, 2, 3, ...,$ suppose that $\sum_{j=1}^{\infty} |a_{ij}| = b_i$ $(i = 1, 2, 3, ...)$ and $\sum b_i$ converges. Then prove that $\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} a_{ij} = \sum_{j=1}^{\infty}\sum_{i=1}^{\infty} a_{ij}$.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Show that every k – cell is compact.

17. Suppose f and g are defined on $[a, b]$ and are differentiable at a point $x \in [a, b]$. Then $f + g$, fg and f/g are differentiable at x, and