8. (a) Show that $y(x) = 2 - x$ is a solution of the integral equation $\int_0^x e^{x-1}\, y(t)\, dt = e^x + x - 1$.

Or

(b) Convert the differential equation $y''(x) - 3y'(x) + 2y(x) = 5\sin x$, $y(0) = 1$, $y'(0) = -2$ into an integral equation.

9. (a) Establish the relation between the differential equation and integral equation.

Or

(b) Show that the integral equation

$$y(x) = \int_0^x (x + t)\, y(t)\, dt + 1$$ is equivalent to the differential equation

$$y''(x) - 2xy'(x) - 3y(x) = 0,\ y(0) = 1,$$

$$y'(0) = 0.$$

10. (a) State and prove Hilbert-Schmidt theorem.

Or

(b) Show that $y(x) = 1$ is a solution of the Fredholm integral equation

$$y(x) + \int_0^1 x\left(e^{tx} - 1\right) y(t)\, dt = e^x - x.$$

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(For the candidates admitted from 2008–2009 onwards)

M.Sc. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2015.

Third Semester

Mathematics

CALCULUS OF VARIATIONS AND INTEGRAL EQUATIONS

Time : Three hours Maximum : 75 marks

SECTION A — (5 × 5 = 25 marks)

Answer ALL questions.

1. (a) Solve the Brachistochrone problem.

Or

(b) Find the extremizing function for

$$J[z(x, y)] = \iint_D \left[\left[\frac{\partial^2 z}{\partial x^2}\right]^2 + \left[\frac{\partial^2 z}{\partial y^2}\right]^2 + 2\left[\frac{\partial^2 z}{\partial x \partial y}\right] - 2zf(x, y)\right] dxdy$$

Where $f(x, y)$ is a known function.

2. (a) Find the extremum of the functional

$$I = \int_{x_1}^{x_2} \left(y'^2 + z'^2 + 2yz\right) dx \quad \text{with} \quad y(0) = 0,$$

$z(0) = 0$ and the point (x_2, y_2, z_2) moves over the fixed plane $x = x_2$.

Or

(b) Show that the curve which extremize the function

$$I = \int_0^{\pi/4} \left(y'^2 - y^2 + x^2\right) dx$$ under the conditions

$y(0) = y'(0) = 1, \quad y\left(\frac{\pi}{4}\right) = y'\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$ is $y = \sin x$.

3. (a) Find the eigen values of the homogenous integral equation $y(x) = \lambda \int_0^1 (3x - 2)t\, y(t)\, dt$.

Or

(b) Solve the boundary value problem $y' + y + x = 0$, $(0 \le x \le 1)$, $y(0) = y(1) = 0$.

4. (a) Explain briefly the types of Kernals through an example.

Or

(b) Solve the Fredholm integral equation

$$y(x) = 1 + \lambda \int_0^1 (1 - 3xt)\, y(t)\, dt.$$

5. (a) Define the Hilbert space and orthogonal system of function. Give an example.

Or

(b) Solve the Volterra integral equation

$$y(x) = 1 + x - \int_0^x y(t)\, dt.$$

SECTION B — (5 × 10 = 50 marks)

Answer ALL questions.

6. (a) Derive Euler-Poisson equation.

Or

(b) Solve the equilibrium problem of a membrane.

7. (a) Find the shortest distance between the parabola $y = x^2$ and the straight line $x - y = 5$.

Or

(b) State and prove Hamilton's principle and hence derive the equation of vibration of a rectilinear bar.