(7 pages)

S.No. 3042

19PMA09

(For the candidates admitted from 2019-2020 onwards)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2020.

Third Semester

Mathematics

TOPOLOGY

Time : Three hours Maximum : 75 marks

SECTION A — (15 × 1 = 15 marks)

Answer ALL questions.

1. For a subset A of a topological space X, the intersection of all closed sets containing A is the ______________.

   (a) closure of A (b) interior of A

   (c) limit point of A (d) neighborhood of A

2. If A is open, $A = Int\, A$; while if A is closed then A ______________.

   (a) A (b) 0

   (c) 1 (d) $\overline{A}$

3. Limit point is otherwise called as ______________ point.

(a) singular (b) cluster

(c) neighbour (d) none of these

4. The metric $\overline{d}$ is called the standard bounded metric corresponding to d then $\overline{d}(x, y) =$ ________.

(a) $\max\{d(x,y),1\}$ (b) $\sup\{d(x,y),1\}$

(c) $\min\{d(x,y),1\}$ (d) none of these

5. The norm of x by the equation is define by ______________.

(a) $\|x\| = \left(x_1^2 + \ldots + x_n^2\right)$

(b) $\|x\| = \left(x_1^2 + \ldots + x_n^2\right)^{1/2}$

(c) $\|x\| = \left(x_1^2 - \ldots - x_n^2\right)$

(d) $\|x\| = \left(x_1^2 - \ldots - x_n^2\right)^{1/2}$

6. If $f : X \to Y$ is a bijection, where X and Y are topological spaces and if both f and $f^{-1} : Y \to X$ are continuous, then f is called a

(a) homeomorphism

(b) homomorphism

(c) isomorphism

(d) meromorphism

7. If $f:[a,b] \to R$ is continuous then there exists an element $c \in [a,b]$ such that $f(x) \le f(c)$ for every $x \in [a,b]$ is statement of ———————— theorem.

(a) intermediate

(b) maximum value

(c) uniform continuity

(d) disjoint

8. If for every open set U of a space X, each component of U is open in X, then X is

(a) connected

(b) path connected

(c) locally connected

(d) locally path connected

9. The image of a connected space under a continuous map is

(a) continuous (b) connected

(c) compact (d) disjoint

10. The product of finitely many compact spaces is ————————.

(a) compact (b) connected

(c) continuous (d) closed

11. If every open covering $\mathcal{A}$ of X contains a finite sub collection that also covers X, then X is said to be ________.

(a) connected (b) continuous

(c) cover (d) compact

12. Every compact subspace of a Hausdorff space is

(a) compact (b) connected

(c) continuous (d) closed

13. Every compact metrizable space X has a ________.

(a) countable basis (b) sub-basis

(c) separation (d) cover

14. A space having a countable dense subset is said to be ________.

(a) First countable

(b) Second countable

(c) Separable

(d) Completely regular

15. Every metrizable space satisfies ________.

(a) second axiom of countability

(b) Lindel of condition

(c) first axiom of countability

(d) basis conditions

SECTION B — ($2 \times 5 = 10$ marks)

Answer any TWO questions.

16. If **B** is a basis for the topology of X and **C** is a basis for the topology of Y, then show that the collection $D = \{B \times C : B \in B \text{ and } C \in C\}$ is a basis for the topology of $X \times Y$.

17. State and prove the pasting lemma.

18. Prove that a space X is locally connected if and only if for every open set U of X. each component of U is open in X.

19. State and prove the extreme value theorem.

20. Let X be a topological space. Let one-point sets in X be closed then prove that X is regular if and only if given a point x of X and a neighborhood U of x, there is a neighborhood V of x such that $\overline{V} \subset U$.

SECTION C — ($5 \times 10 = 50$ marks)

Answer ALL questions.

21. (a) Let X be a topological space. Then prove that the following conditions hold. (i) ϕ and X are closed (ii) Arbitrary intersections of closed sets are closed (iii) Finite unions of closed sets are closed.

Or

(b) If $\mathfrak{B}$ is the basis for the topology of X and $\mathcal{C}$ is a basis for the topology of Y. then show that the collection $\mathfrak{D} = \{B \times C \mid B \in \mathfrak{B}$ and $C \in \mathcal{C}\}$ is a basis for the topology of $X \times Y$.

22. (a) Let X and Y be topological spaces; let $f : X \to Y$. Then show that the following are equivalent:

(i) f is continuous.

(ii) For every subset A of X one has $f(\overline{A}) \subset \overline{f(A)}$.

(iii) For every closed set B of Y, the set $f^{-1}(B)$ is closed in X

(iv) For each $x \in X$ and each neighborhood V of $f(x)$, there is a neighborhood U of x such that $f(U) \subset V$.

Or

(b) Prove that the topologies on R^n induced by the Euclidean metric d and the square metric ρ are the same as the product topology on R^n.

23. (a) State and prove the intermediate value theorem.

Or

(b) Prove that a finite Cartesian product of connected space is connected.

24. (a) State and prove Lebesgue number lemma.

Or

(b) Prove that the Product of finite many compact spaces is compact.

25. (a) State and prove Urysohn lemma.

Or

(b) State and prove Tietz extension theorem.

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